

Calculation of the One W Loop $H \rightarrow \gamma\gamma$ Decay Amplitude with a Lattice Regulator

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There has been a controversial recent claim that the standard result on the Higgs to two photon decay rate is incorrect, with the use of dimensional regularization fingered as the alleged culprit. Given the great importance of the $H \rightarrow \gamma\gamma$ process as a possible Standard Model Higgs discovery channel at the LHC if the Higgs mass is light, it is critical to find a way to check the correctness of the results of dimensional regularization for this process. Here we report the results of a perturbative calculation of the $H \rightarrow \gamma\gamma$ decay amplitude using a spacetime lattice as a UV regulator, which is the only known gauge-invariant regulator for non-Abelian gauge theories other than dimensional regularization. We find that the decay amplitude calculated using lattice-regularized perturbation theory is consistent to very high statistical accuracy with the decay amplitude obtained using dimensional regularization.

The decay of the Higgs boson into two photons, $H \rightarrow \gamma\gamma$, is a crucial channel in the search for light Higgs bosons at the Large Hadron Collider (LHC) (see e.g. [1]) especially if the Higgs mass is close to the LEP bound of $M_H \gtrsim 114.4 \text{ GeV}/c^2$ [2]. In the Standard Model (SM) there is no direct vertex for this decay, and the process proceeds through loops of intermediate gauge bosons and quarks. At one-loop level, the two most important contributions to the decay amplitude are from the top quark loop and from W^\pm boson loops. The evaluation of the latter contribution has recently been questioned. The W^\pm -loop contribution to the $H \rightarrow \gamma\gamma$ decay amplitude takes the form

$$\mathcal{M}_{\mu\nu} = -\frac{e^2 g}{8\pi^2 M_W} [k_\mu^1 k_\nu^2 - g_{\mu\nu} (k^1 \cdot k^2)] \frac{F(\tau)}{\tau}, \quad (1)$$

where e and g are the $U(1)_{EM}$ and $SU(2)_L$ couplings respectively, M_W is the W -boson mass, $k_\mu^{1,2}$ are the four-momenta of the emitted photons, $\tau = M_H^2/4M_W^2$, and M_H is the mass of the Higgs boson. The function $F(\tau)$ was first calculated in [3, 4] many years ago, with the result

$$F(\tau) = \frac{3}{2} + \tau + \frac{3}{2}(2 - \tau^{-1})[\arcsin \sqrt{\tau}]^2, \quad (2)$$

for $\tau \leq 1$. These calculations used dimensional regularization, although [4] also derived some beautiful low-energy theorems on the properties of the amplitude which did not explicitly call on any particular regularization procedure.

Recently this amplitude has been examined again by Gastmans, Wu, and Wu [5]. Ref. [5] carried out the calculation in unitary gauge directly in four dimensions without any use of dimensional regularization; they used a particular momentum-routing prescription to handle the divergences arising at intermediate stages in the calculation of the amplitude. The result they obtained is

$$F(\tau) = \frac{3}{2} + \frac{3}{2}(2 - \tau^{-1})[\arcsin \sqrt{\tau}]^2, \quad (3)$$

which differs from the classic result by the absence of a term linear in τ . This is a highly surprising result, since one expects that the calculation of a physical amplitude in quantum field theory must not depend on the regularization prescription. This difference was blamed on a problem with dimensional regularization. The claim was that dimensional regularization assigns a finite value to a contribution to the amplitude which is zero in $d = 4$ when evaluated using the prescriptions of [5]. If the striking result of [5] were the correct form of the decay amplitude, it would throw into question a vast number of calculations carried out with dimensional regularization. Furthermore, Eq. (3) gives a significantly smaller decay rate than Eq. (2) for Higgs masses where the diphoton decay is an important discovery channel. For example, for $M_H = 115 \text{ GeV}/c^2$ the decay width would be reduced by $\approx 50\%$ [5]. If correct, this would have significant implications for Higgs searches at the LHC.

Given the long experience with the reliability of dimensional regularization, the claim of [5] must naturally be viewed with skepticism, and a number of papers [6] have appeared criticizing the results of Gastmans *et al.* from a variety of perspectives. However, given the extraordinary phenomenological importance of correctly determining this amplitude in the Standard Model, it is highly desirable to add an extra check by explicitly computing $M_{\mu\nu}$ using a reliable gauge-invariant regulator other than dimensional regularization.

Lattice as a regulator — The list of reliable gauge-invariant regulators for non-Abelian gauge theories is very short. Naive momentum cutoffs break gauge invariance, while Pauli-Villars-like regulators have serious problems in non-Abelian theories (see e.g. [7]). In perturbative gauge theory calculations dimensional regularization [8] is omnipresent since it manifestly preserves gauge invariance and is easy to use. The only other known gauge-invariant regulator is a spacetime lattice, which has the effect of imposing a momentum cutoff $\sim 1/a$, where a is the lattice spacing, but does this in

a far more subtle way than a naive momentum cutoff, with manifest gauge invariance at any a . Lattice regularization has the great advantage that it can be used beyond perturbation theory, providing a nonperturbative definition of a theory. This has led to much progress in the nonperturbative understanding of gauge theories, e.g. using Monte Carlo methods to compute the full lattice-regularized Euclidean path integral of QCD. A lattice regulator can also be used in perturbative calculations, where it gives an alternative route to obtaining results that otherwise could only be obtained using dimensional regularization.

Lattice regulators have two chief disadvantages for perturbative calculations compared to dimensional regularization. The first disadvantage is that a lattice regulator breaks rotational and translation invariance, keeping only discrete subgroups of these symmetries. Rotational symmetry is crucial for enabling analytic evaluation of loop integrals in perturbative calculations; the loss of these symmetries on the lattice at finite a means that loop integrals must be evaluated numerically. This is certainly a practical inconvenience, but it is not a problem of principle. The second disadvantage is of a deeper nature. It is not known how to implement lattice regulators for chiral gauge theories; for some recent reviews see [9]. Since the Standard Model is a chiral gauge theory this precludes the use of lattice regulators for computing most SM observables sensitive to the electroweak sector.

Fortunately the difficulties with chiral fermion couplings do not play a role here. This is because only the W^\pm -mediated contribution to the $H \rightarrow \gamma\gamma$ decay amplitude has been questioned, not the top-quark-mediated contribution. Using the lattice regulator we can simply compute the $H \rightarrow \gamma\gamma$ decay amplitude with all the fermion fields turned off.

Setup — We will carry out our calculations below the threshold for the $H \rightarrow W^+W^-$ decay ($\tau < 1$) since the LHC has already excluded SM Higgs boson masses $m_H \gtrsim 146$ GeV [10] at $> 95\%$ confidence. This assumption makes the Wick rotation to Euclidean space especially simple. (Above threshold, the calculation could also be done using lattice perturbation theory [11].) To evaluate the decay amplitude, we go to Euclidean space, choose $\mu = \nu = 1$, and work in the Higgs boson rest frame, so that $k_1 = (iM_H/2, 0, 0, M_H/2)$, $k_2 = (iM_H/2, 0, 0, -M_H/2)$ for on-shell photons. The amplitude then takes the form

$$\mathcal{M}_{11} = -\frac{e^2 g M_W}{4\pi^2} F(\tau). \quad (4)$$

The lattice calculation proceeds by calculating $F(\tau)$ numerically at a range of values of the lattice spacing a , taking the continuum limit $M_W a, M_H a \rightarrow 0$ with τ held fixed. With this end in view we calculate the dimensionless function $F(\tau, aM_W)$, where the dependence on aM_W is due to lattice artifacts, and perform a two dimensional

fit in τ and aM_W to recover continuum limit, $aM_W \rightarrow 0$, giving $F(\tau) \equiv F(\tau, 0)$. For the fit function we take as our guide the functional form obtained from current analytic calculations and find it sufficient to consider

$$F(\tau, aM_W) = c_1 + c_2 \tau + c_3 \left(2 - \frac{1}{\tau}\right) [\arcsin \sqrt{\tau}]^2, \quad (5)$$

where c_1, c_2, c_3 are functions of aM_W .

For the gauge boson action that we use, a simple parametrization of the aM_W dependence of $c_1(aM_W)$ is found to be sufficient and c_2 and c_3 appear constant for the values of aM_W that we choose.

Lattice Feynman rules — As in any perturbative calculation of an amplitude, we must fix a gauge to proceed, and we choose to use unitary gauge. The lattice action for the gauge and Higgs sector of Standard Model in unitary gauge is described in Ref. [12]. To carry out the calculation we use the HiPPY and HPSRC packages developed for automated lattice perturbation theory [13, 14]. We use the Symanzik-improved lattice action [15] for the vector-boson vertices that arise from the pure $SU(2)$ gauge part of the action. The improvement is important since this greatly reduces lattice artifact contributions. These vertices are available in the automated packages and reduce to the continuum ones in the $a \rightarrow 0$ limit. For the W boson propagator we add a mass term to the quadratic part of the gauge boson action and invert numerically to give the lattice version of the Proca propagator, as is appropriate to unitary gauge.

The vector-boson mass term arises from the lattice Lagrangian [12]

$$L_H = -\frac{1}{2} \sum_{\mu=1}^4 [av + aH(\mathbf{x} + a\mathbf{e}_\mu)][av + aH(\mathbf{x})] \\ \times \text{Tr}[e^{gaT_i W_{i\mu}(\mathbf{x})} e^{-g'aT_3 B_\mu(\mathbf{x})}] \quad (6)$$

where v is the vacuum expectation value of the radial scalar field, $H(\mathbf{x})$ is the Higgs field, $W_i(\mathbf{x})$, $i = 1, 2, 3$ are the $SU(2)_L$ gauge boson fields and $B(\mathbf{x})$ is the $U(1)_Y$ gauge field. The T_i are the anti-hermitian generators of $SU(2)$ satisfying $[T_i, T_j] = -\epsilon_{ijk} T_k$. As is usual for lattice actions L_H is dimensionless. The term quadratic in the W and B fields generates the W^\pm and Z mass terms with $M_W = M_Z = gv/2$ and where

$$B_\mu = s_W Z_\mu + c_W A_\mu, \quad W_{3\mu} = -c_W Z_\mu + s_W A_\mu. \quad (7)$$

Here $s_W \equiv \sin \theta_W$, $c_W \equiv \cos \theta_W$, where θ_W is the Weinberg angle, $\tan \theta_W = g'/g$. This term also contains the usual HW^+W^- vertex and, because the lattice spacing is non-zero, it additionally gives rise to extra interaction terms on the lattice which have no continuum counterpart. It should be emphasized that when inserted into a Feynman diagram these extra terms give contributions which are non-zero in the continuum limit; indeed, such

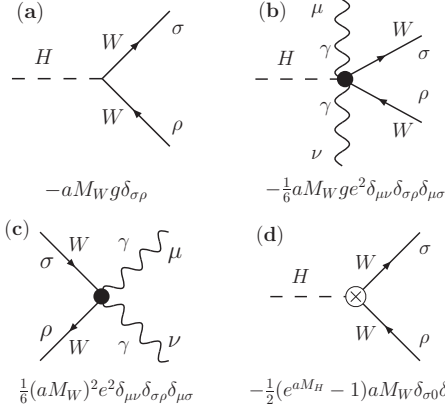
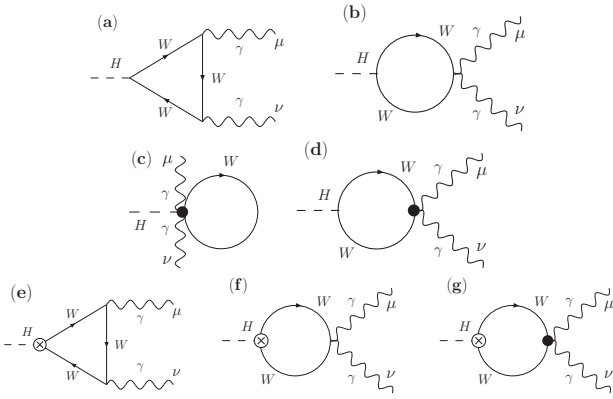
FIG. 1. Feynman rules derived from L_H (6).

FIG. 2. Feynman diagrams (see text for description).

contributions are a necessary consequence of maintaining gauge invariance while regularizing the theory by restricting momenta to lie in the lattice Brillouin zone. These extra Feynman rules are derived by expanding the trace term in L_H , using the Baker-Campbell-Hausdorff formula to the appropriate order and using (7) to identify the photon field A_μ . The HW^+W^- and additional Feynman rules are displayed graphically in Fig. 1. The rule in (d) arises because of the presence of $H(\mathbf{x} + a\mathbf{e}_\mu)$ in L_H , and, because the Higgs boson is at rest, the Lorentz index on the W -bosons attached to this vertex is restricted to $\mu = 0$ as shown in the figure.

Summary of diagrams — In our unitary-gauge lattice-regulated calculation there are seven Feynman diagrams for the decay process, shown in Fig. 2. In the triangle (a) and turnip (b) diagrams, the vector boson vertices arise from the pure $SU(2)$ gauge vector boson action and are similar to ones that arise in QCD. The ankh (c) and lattice turnip (d) diagrams arise from the additional lattice Feynman rules. Diagrams (e), (f) and (g) are modifications of (a), (b) and (d) to include the lattice forward derivative ∇_μ^+ coupling due to the $(\nabla_0^{(+)}H)W^+W^-$ term in the action. Note that there is no such modification

of the ankh diagram as it would require the photons to have Lorentz index $\mu = 0$, which they do not.

Cancellation of divergences — The result of combining these diagrams must be finite since there is no tree-level process for $H \rightarrow \gamma\gamma$, so no counter-term can be constructed to cancel any remaining divergences through the usual renormalization prescription. Therefore divergences $(aM_W)^{-n}$ present in individual diagrams must cancel. To investigate this cancellation in more detail, we fix τ and study the dependence on aM_W . By verifying the cancellation of divergences and obtaining a finite answer for the matrix element, \mathcal{M}_{11} , we perform a very strong check on the calculation. Diagram (a) potentially has a $(aM_W)^{-6}$ divergence but this vanishes trivially. Both (a) and (b) have $(aM_W)^{-2}$ and $(aM_W)^{-4}$ divergences. The $(aM_W)^{-4}$ divergences must cancel between the two diagrams, since no such divergences appear in any other diagram in Fig. 2. This is confirmed numerically, and we also find that the $(aM_W)^{-2}$ divergence cancels between these two diagrams. The lattice diagrams (c) and (d) contain $(aM_W)^{-2}$ divergences which cancel between them. This is a good check for the derived lattice Feynman rules. The derivative diagrams (e)-(g) contain modifications of the HW^+W^- vertex which restricts the Lorentz index on the W bosons at this vertex to be $\mu = 0$. They each clearly contain a $(aM_W)^{-2}$ divergence, and we find that the contributions from all three diagrams are needed for cancellation. This acts as another strong check on the lattice Feynman rules since the contribution from (g) is needed to cancel the divergences of (e) and (f), thus linking the rule generating diagram (d) with the standard lattice rules which give rise to diagrams (a) and (b). Indeed, we find numerically that diagrams (e), (f) and (g) cancel exactly.

τ dependence — We deduce $F(\tau)$ by performing a simultaneous fit in (τ, aM_W) to $F(\tau, aM_W)$ so that the continuum extrapolation $a \rightarrow 0$ can be done. We initially fit the sum of the contributions from diagrams (a)+(b) and from (c)+(d) separately, denoting these $F^{(a+b)}$ and $F^{(c+d)}$, respectively. For each fixed τ value we compute $F(\tau, aM_W)$ in both cases for values of aM_W $0.003 \leq aM_W \leq 0.2$. We find that $F^{(a+b)}(\tau, aM_W)$ has no discernible aM_W dependence in this range at fixed τ and we find the fit result

$$F^{(a+b)} = 2.088(8) + 0.660(17)\tau + 1.500(3) \left(2 - \frac{1}{\tau}\right) [\arcsin \sqrt{\tau}]^2 \quad (8)$$

with $\chi^2 = 0.25$. There is no improvement in fit from including lattice artifact aM_W polynomials or logarithms. For illustration, we show in Fig. 3 the data for the smallest aM_W value, $aM_W = 0.003$, and the best fit curve in (8).

The contribution of the lattice-induced diagrams, (c) and (d), does have a dependence on lattice artifacts and

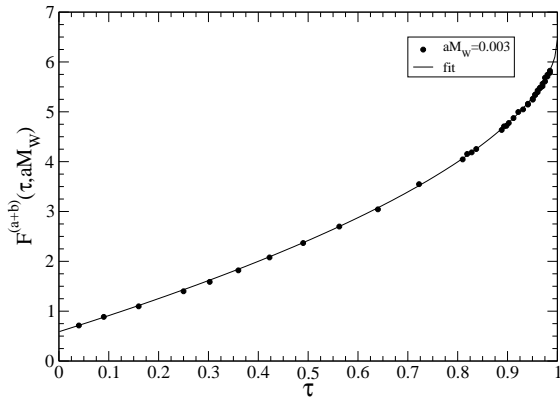


FIG. 3. A plot for illustration of $F^{(a+b)}(\tau, aM_W)$ for $aM_W = 0.003$ and the fit from Eq. (8) for $F^{(a+b)}(\tau)$. There are no observable lattice artifact effects for any value of aM_W used.

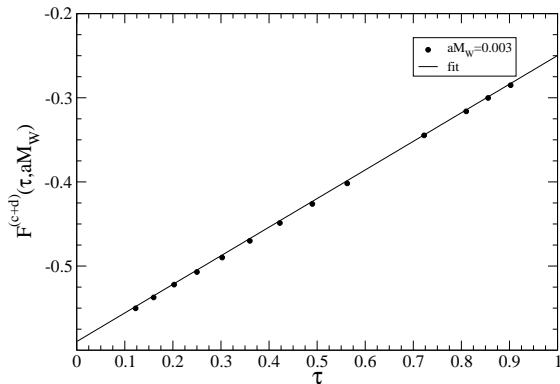


FIG. 4. A plot for illustration of $F^{(c+d)}(\tau, aM_W)$ for $aM_W = 0.003$ and the fit from Eq. (9) for $F^{(c+d)}(\tau)$. While there is a lattice artifact term in the fit, its effect even for the largest value of aM_W used is not discernible on the plot being a correction of order 1%

is well-described by the function

$$F^{(c+d)} = -0.590(1) + 0.340(1)\tau - 0.24(3)(aM_W)^2 \log aM_W, \quad (9)$$

with $\chi^2 = 0.65$. The fits indicate that there is no need to include a polynomial in aM_W and that the lattice artifact term turns out to be independent of τ . In Fig. 4 we show the data for the smallest aM_W value, $aM_W = 0.003$, and the best fit curve in Eq. (9). For the values of aM_W used we do not illustrate the lattice artifact dependence by plotting the data for other values of aM_W since for the largest value used the correction is of order 1% and unresolvable on the scale of the graph shown. The well-controlled continuum extrapolation is, to a high degree, due to the use of the Symanzik improved gauge action. The derivative diagrams, (e)-(g), cancel exactly, and give no net contribution. For the graphs shown the error bars are smaller than the symbol size.

From Eq. (8) and (9) we see that $F^{(a+b)}(\tau)$ contains the arcsin term which gives rise to the threshold behaviour,

as we would expect, and the additional lattice terms in $F^{(c+d)}(\tau)$ give a simple linear behaviour in τ which is, however, vital to the question in hand namely the value of c_2 . We note also that neither $F^{(a+b)}(\tau)$ or $F^{(c+d)}(\tau)$ individually vanish at $\tau = 0$.

A fit to the contributions of all four diagrams (a) - (d) gives the final result in the continuum limit of

$$F(\tau) = 1.498(8) + 1.000(17)\tau + 1.500(3) \left(2 - \frac{1}{\tau}\right) [\arcsin \sqrt{\tau}]^2, \quad (10)$$

which is in strong agreement with the established dimensional regularization result. We note the fact that $F(0) = 0$ arises directly from our calculation, as it does in dimensional regularization, without recourse to the Dyson subtraction of other calculations in $D = 4$ [5].

Conclusions — Physical predictions should not depend on the particular way a quantum field theory is regulated. This paper shows how perturbative lattice calculations can be used to check the correctness of continuum calculations of observables, so long as the questions involved do not directly involve couplings of gauge fields to complex-representation Weyl fermions. Agreement between the lattice calculation here of the W -mediated contribution to $H \rightarrow \gamma\gamma$ and the longstanding result obtained with dimensional regularization should put to rest any controversy regarding the Standard Model prediction for this decay channel.

Acknowledgments — We are very grateful to B. C. Allanach, T. Ishii, and S. Lee for helpful discussions, and especially to A. Maharana for collaboration in the early stages. This work was supported by STFC under grant ST/G000581/1. The calculations for this work were, in part, performed on the University of Cambridge HPCs as a component of the DiRAC facility jointly funded by STFC and the Large Facilities Capital Fund of BIS.

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